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INDIRECT FIRE APPRECIATION MODELLING

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NORTH ATLANTIC TREATY ORGANIZATION (NATO)

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NATO LETTER OF PROMULGATION

16 June 2021

1. The enclosed Allied Ordnance Publication AOP-4654, Edition A, Version 1, INDIRECT FIRE APPRECIATION MODELLING, which has been approved by the nations in the NATO ARMY ARMAMENTS GROUP (NAAG), is promulgated herewith. The agreement of nations to use this publication is recorded in STANAG 4654.

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4. This publication shall be handled in accordance with C-M(2002)60.

Zoltán GULYÁS
Brigadier General, HUNAF
Director, NATO Standardization Office

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RECORD OF RESERVATIONS

CHAPTER	RECORD OF RESERVATION BY NATIONS
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RECORD OF SPECIFIC RESERVATIONS

[nation]	[detail of reservation]
USA	Page 1-1, Paragraph 1.1. - The USA recognizes that effects are neither “achieved” nor “delivered”. Effects are rather “created”, as shown in higher level allied joint publications.
Note: The reservations listed on this page include only those that were recorded at time of promulgation and may not be complete. Refer to the NATO Standardization Document Database for the complete list of existing reservations.	

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CHAPTER 1 GENERAL

1.1. INTRODUCTION

1. Fire appreciation is a logical process of reasoning by which a commander considers all relevant types and optimal quantities of available fires to decide what effect can be achieved against a particular target. The ultimate goal is answering five essential questions related to the engagement of a target with a certain delivery effect: which Weapons (Who?) will fire how many rounds (How Many?) of what type of munition (What?) using which firing method (How?) and which aimpoints (Where?).

1.2. AIM

1. The principal aim of this agreement is to standardise the mathematical models for indirect fire appreciation.

1.3. AGREEMENT

1. Participating nations agree to use the NATO Indirect Fire Appreciation Models as part of the targeting and engagement process using indirect fire.

2. This agreement permits flexibility by accommodating certain specific national target damage methodologies, aimpoint policies and measures of target vulnerability.

1.4. RELATED DOCUMENTS

1. Related documents:
   - STANAG 4635 - NATO Armaments Error Budget Model
   - STANAG 4355 - Modified Point Mass Trajectory Model and Five Degree of Freedom Trajectory Model
   - STANAG 4537 - NATO Armaments Ballistic Kernel (Including AOP-37 & AOP-49)
   - Feasibility Study for the Development of an Indirect Fire Appreciation Model (distributed by BEL to all members AC/225(LG/4-SG/2) by e-mail on 02 Jul 04).

1.5. DEFINITIONS AND SYMBOLS

1. Definition of terms and symbols used in this publication are provided in Annex A (Definition of Terms) and Annex H (List of Symbols).

1.6. DETAIL OF THE AGREEMENT

1. The details of the agreement are provided in the following Annexes:

   A. Definition of Terms
   B. Mathematical Models to Determine Lethality
   C. Modelling Calculation of Delivery Errors
   D. Target Behaviour
   E. Aimpoint Placement
   F. Mathematical Model of Delivery Accuracy
   G. Damage Threshold Contours
   H. List of Symbols

2. Each of the lethality models determines the probability of kill for a single target element due to either a munition or submunition. The target element location is provided as input for the lethality function. Prior to using the lethality models either the reference point of dispense of a directed energy munition or the functioning point for other munitions is determined. Figure 1.1 depicts a typical encounter for a fragmenting munition.

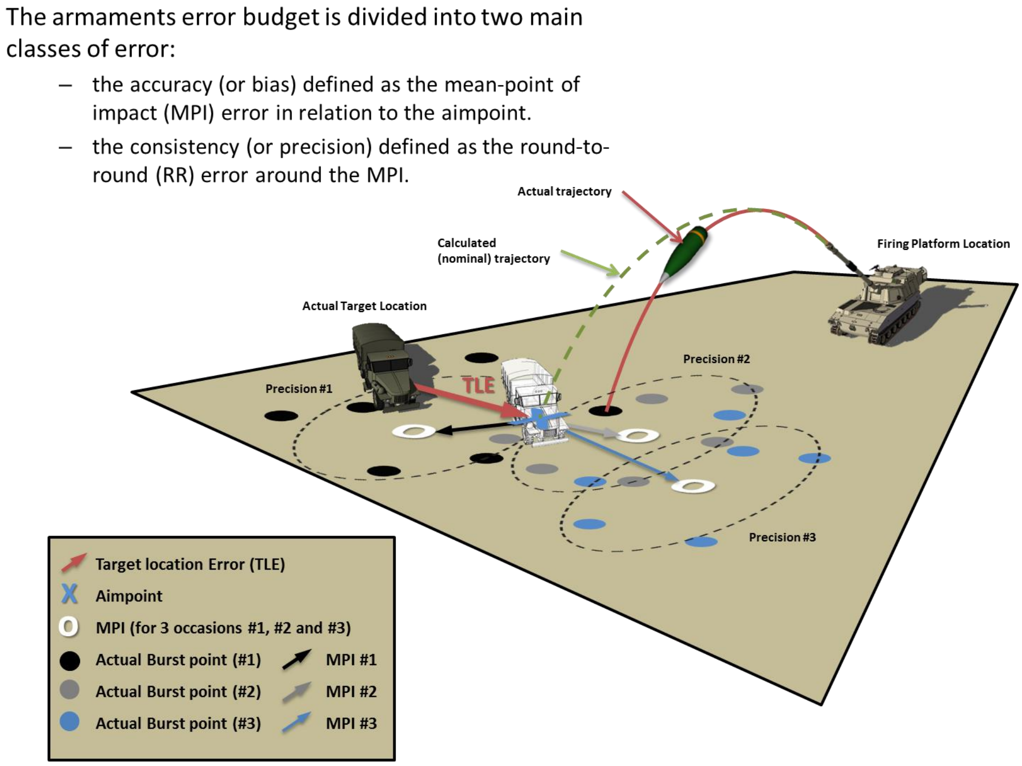

Figure 1.1. Typical scenario for a fragmenting munition including firing platform location, target location, aimpoint, burst point, and all associated errors.

3. The general mathematical model for the delivery errors can be found in STANAG 4635. Additional requirements for modelling lethality can be found in Annex B.

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ANNEX A DEFINITION OF TERMS

A.1. GENERAL

1. The technical terms defined in this AOP-4654 are intended for use within the context of Indirect Fires Appreciation modeling.

2. The definitions in this Annex as prepared by the NIFAK TOE have been compared with the definitions in the NATO documents AAP-6 (Edition 2014) "NATO GLOSSARY OF TERMS AND DEFINITIONS", the AAP-39 (2012) "NATO HANDBOOK OF LAND OPERATIONS TERMINOLOGY" and the GLOSSARY section of the US Field Manual FM 6-40 (1999).

A.2. DEFINITION OF TERMS

1. Aimpoint

The Aimpoint is the location at which a round is aimed.

2. Aimpoints Area

The area which is used to compute all the aimpoints.

3. Angle of Fall

The angle of fall is the vertical angle at the level point between the line of fall and the base of the trajectory (see Table A.1).[1]

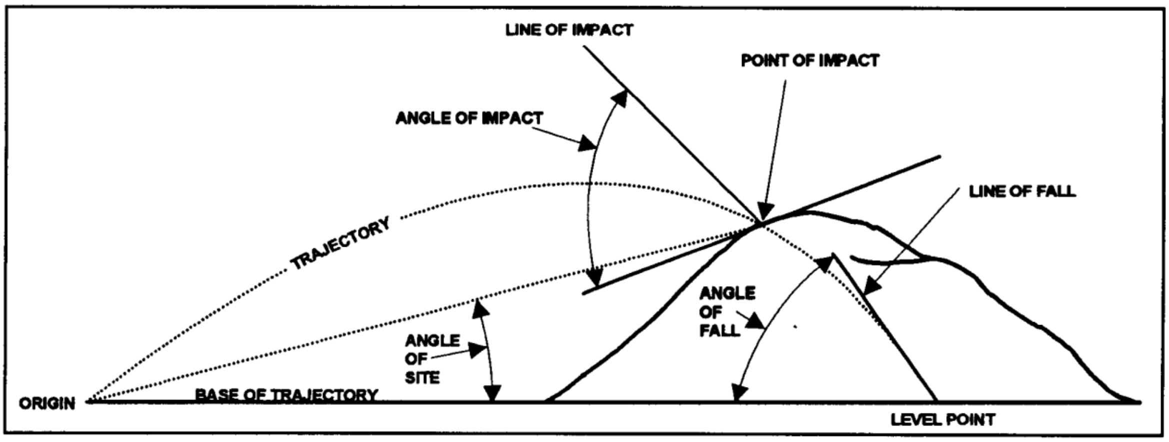

Figure A.1. Terminal Elements of the Trajectory (from FM 6-40).

[1] This is consistent with STANAG 4119 with the exception that it is flat earth.

4. Angle of impact

This is the angle a muniton's trajectory makes with the tangential plain at the point of impact. In the case of airbust munitions, the point of impact is taken as the functioning point (see Table A.1).

5. Point of impact

The point at which a munition first makes contact with an object (see Table A.1).

6. Ancillary Damage

The unintended but desirable damage to personnel and/or objects due to the application of fires.

7. Collateral Damage

The unintended and undesirable effects to personnel and/or objects due to the application of fires.

8. Complex Target

A target consisting of multiple (non-identical) target elements that function together to produce a specific capability. Defeat of a complex target is defined by the damaging of specific combinations of target elements.

9. Damage Function

This is a mathematical function used to calculate the probability of damage to a target element. This is primarily a mathematical function of distance and direction from the functioning point but will also depend on other parameters[2].

10. Damage Matrix

A mathematical matrix containing values of probability of damage for a target element as a function of distance and direction from a functioning point. Changes to any parameter will result in a different damage matrix.

11. Danger Close

Engagement conditions characterized by probability of undesirable damage (risk) to friendly forces in an effort to engage a target to achieve mission objectives.

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[2] The calculated value will depend on many parameters such at target type, munition type and ballistic terminal conditions.

12. False/Unintended Target Element

Any effect or object that could cause seeker munitions to acquire something other than the intended target element. A false target element can be due to spurious returns or clutter.

13. Fractional Damage

The number of damaged target elements within a simple target divided by the total number of target elements within the simple target, normally expressed as a percentage.

14. Fragmenting Munitions

Munitions, whose primary lethal mechanisms are fragmentation and blast. In general, it is not necessary for fragmenting munitions to actually impact on a target element to have a probability of damaging it.

15. Fratricide

Damage to friendly combatants and materiel caused by the mistaken targeting of those combatants or materiel.

16. Functioning Point

The location at which a munition functions. That point is the fuze function point which can be a point of impact, point of burst, or sub-munition dispense.

17. Guided Munitions

Munitions that have some ability to alter their course.

18. Gun-Target Line

An imaginary straight line from gun to target at the time of fire. Related terms: observer-target line; spotting line.

19. Initiation Point

The point (in space and time) at which the warhead effects initiates.

20. Kill Criterion

A nationally pre-defined level of damage which prevents that target element from performing its primary function/mission. A target element is either functional or it is incapacitated (killed) for that criterion.

21. Lethal Area

A measure of the lethality of a specific munition against a specific target element. Mathematically, the lethal area is the integral of the damage function over the ground plane.

$$A_L = \iint_S P_k (x, y)\, dx\, dy \tag{A-1}$$

with:

P_k = probability of target element kill
S = the entire x,y plane
x,y = Cartesian coordinates in a horizontal system

22. Moving Target

The movement of target elements during a mission. This movement is defined by velocity vectors and positions as a function of time.

23. Multiple Rounds Simultaneous Impact (MRSI)

The ability to shoot multiple rounds from one weapon on a target such that the rounds land in the target area at the same time.

24. Posturing Sequencing

The action of personnel changing their posture during a mission which will change vulnerability.

25. Precision Target

A target consisting of target element(s) that has/have a low Target Location Error (TLE). A precision target may be simple or complex.

26. Range-Deflection Coordinates

Cartesian coordinate system with axes parallel to the gun-target line (range) and perpendicular to the gun-target line (deflection).

27. Risk Distance

The distance from the target to a point where the probability of unintended damage to an object (user defined) is always less than a specified threshold value.

28. Risk Estimate Distance (RED)

The distance from the target to a point where the probability of damage to a friendly combatant is always less than a specified threshold value (See section A.3. for a methodology to construct the RED).

29. Seeker Munitions

Munitions that have technology to detect and guide to (or aim at) a target element signature in the terminal phase of its trajectory.

30. Sensor Fuzed Munitions

Munitions that have technology to detect a single target element and initiate a fuze to damage it.

31. Simple Target

A target consisting of one or more identical target elements.

32. Target
The object that should be engaged by the firing mission.

33. Target Area

The defined area within which the target elements are located.

34. Target Element

The smallest entity that defines both simple and complex targets.

35. Target Hardening

The change in a target that results in a change of its vulnerability during a mission.

36. Target Kill

A Target Kill is achieved when the target elements of a target are defeated to satisfy a specific national definition.

37. Unguided Munitions

Munitions that follow a ballistic trajectory.

38. Uniform Target Distribution

The placement of target elements in a simple target using a statistical uniform probability distribution.

39. Volley

A coordinated delivery of a single round from each member of a firing unit.

A.3. METHODOLOGY TO CONSTRUCT THE RED

1. Figure A.2 shows the method from reference [1] to construct the RED, with Freedom of movement (similar to TLE), The Munition Effects Radius (MER), probable errors and the sheaf offset.

2. The MER is defined as a distance from the munition functioning where the Probability of Incapacitation is below a certain threshold value (e.g. 01.) (See Figure A.3). Practically the curve can be cut off at the value were the fragment speed is below the V_{50}. Above the V_{50} velocity the fragments are more likely to penetrate a defined target surface. According to reference [2] the penetration is defined as an event during which a projectile creates a discontinuity in the original surface of the target. Perforation requires that, after projectile or its remnants are removed, light may be seen through the target. Since penetration is a somewhat stochastic event, there is a need to define some statistical parameters. V_{10} is the velocity at which a given projectile will defeat a given target 10% of the time. V_{50} is the velocity at which a given projectile will defeat a given target 50% of the time, and V_{90} is the velocity at which a given projectile will defeat a given target 90% of the time. These quantities are depicted in Figure A.4.

3. For practical purposes the RED can be simplified as:

$$RED = MER + 3 \times PE_R. \qquad \textbf{(A-2)}$$

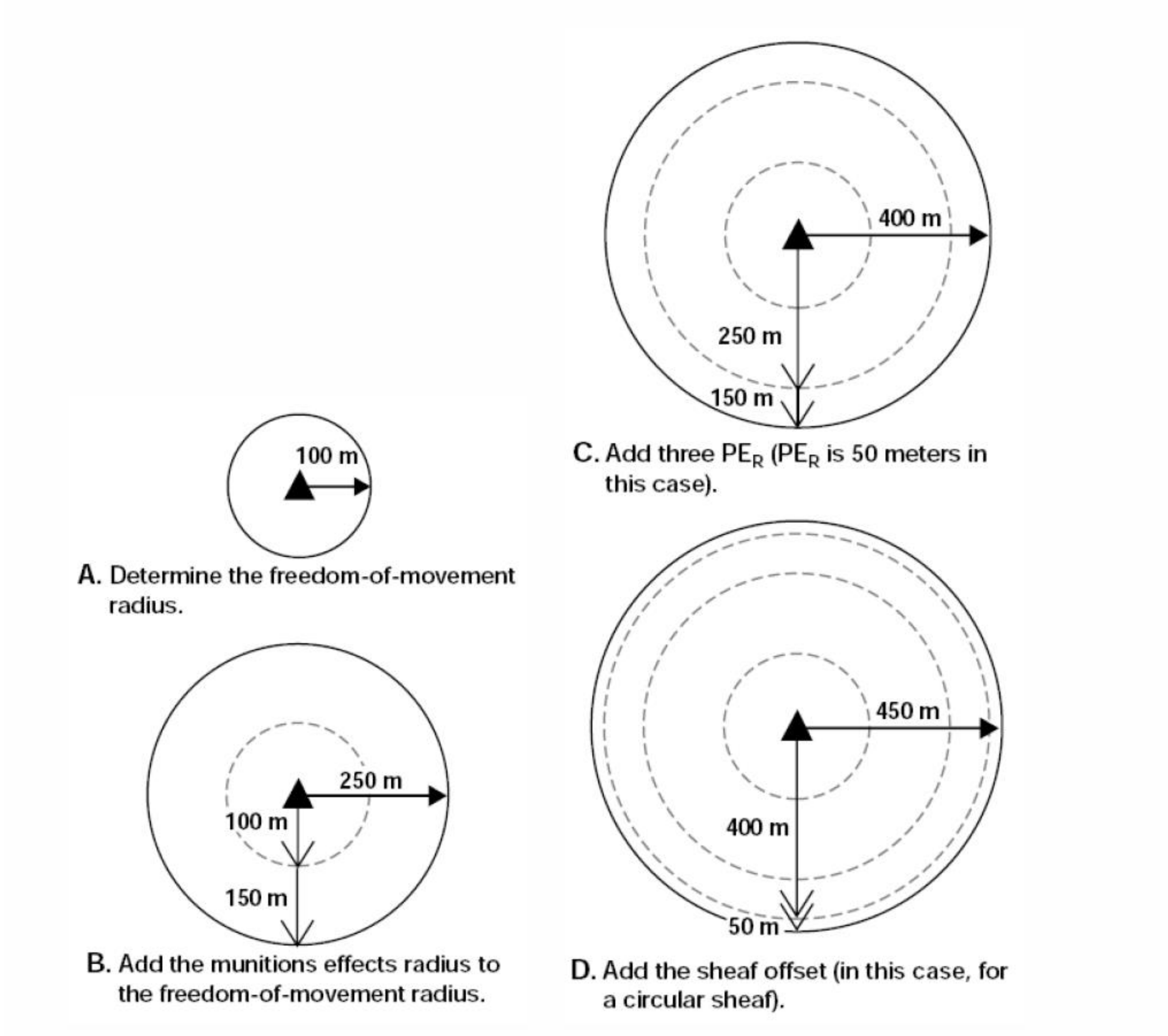

Figure A.2. Method to construct RED.

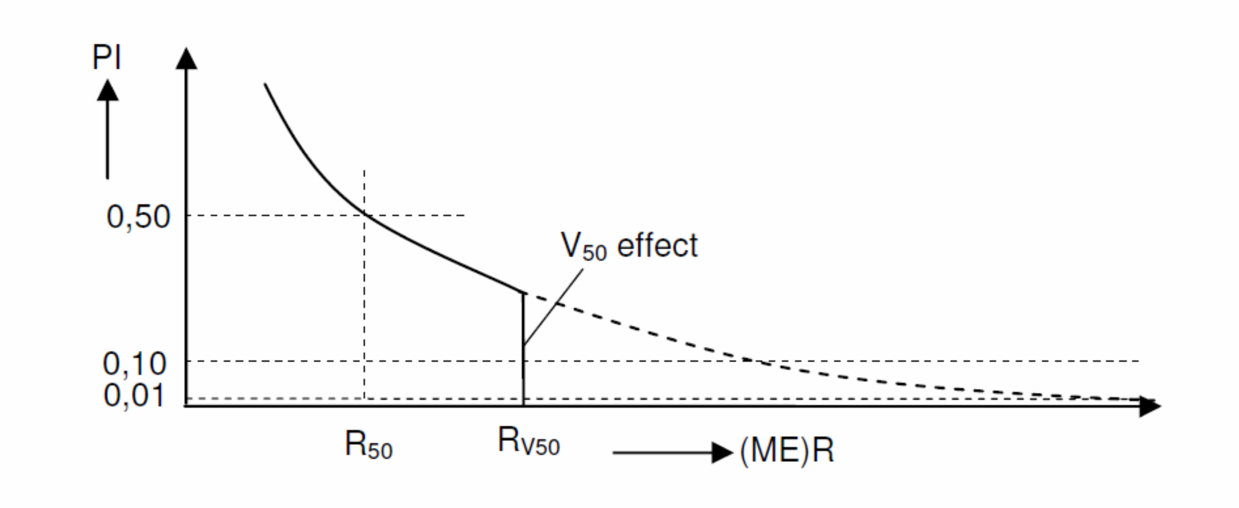

Figure A.3. Example of the Probability of Incapacitation.

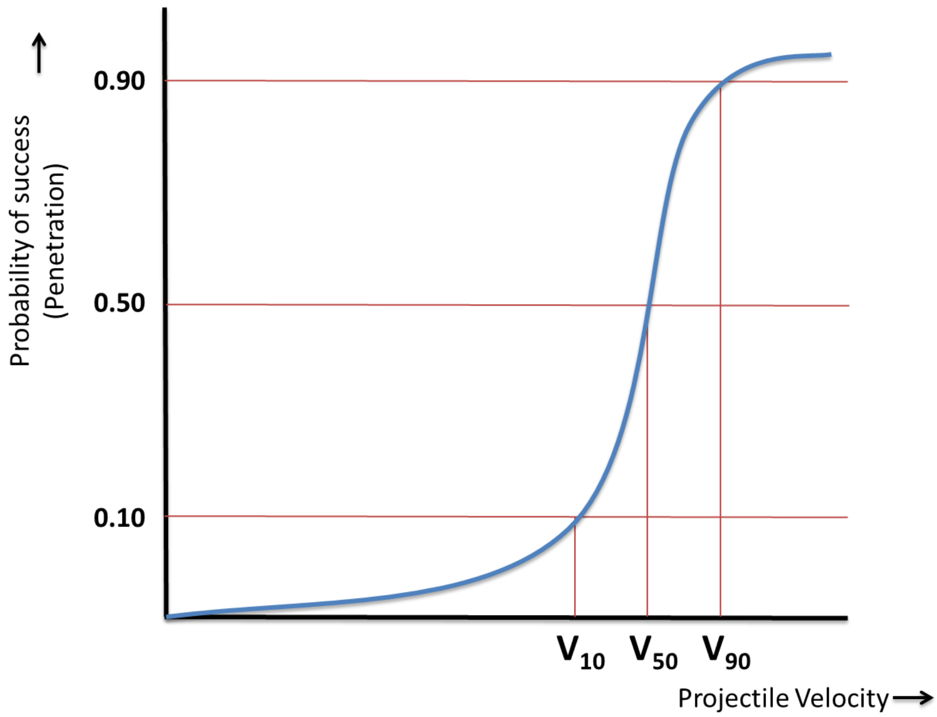

Figure A.4. Statistical Velocities defined.

A.4. REFERENCES

[1] R.L. Lusher “Deliberate NFA sizing for Combat” Field Artillery Journal, March-April 1999, pp 41-45.

[2] Carlucci, Sidney S. Jacobson (2008). Ballistics: Theory and Design of Guns and Ammunition. CRC Press. p. 310.

ANNEX B MATHEMATICAL MODELS TO DETERMINE LETHALITY

B.1. INTRODUCTION

1. This annex contains various mathematical models that are used to determine how many munitions should be fired by calculating the amount of damage from a given quantity of fire. A “Level of Fidelity” is indicated for each model relative only to the other models described in this annex. This annex contains the following:

- Computing the Lethality of Fragmenting Munitions From Physical Properties (Section B.2.)
- The Carleton Damage Function for Unitary Fragmenting Munitions (Section B.3.)
- The Circular Cookie Cutter Damage Function for Unitary Fragmenting Munitions (Section B.4.)
- The Cookie-Cutter Damage Function for a Pattern of Bomblet Munitions (Section B.5.)
- Modelling Individual Bomblet Munitions (Section B.6.)
- Damage Matrix Methodology (Section B.7.)
- Modelling Sensor Fuzed Munitions (Section B.8.)

B.2. A SIMPLIFIED METHOD FOR COMPUTING THE LETHALITY OF FRAGMENTING MUNITIONS BASED ON PHYSICAL PROPERTIES

B.2.1 Introduction

1. This section provides a foundation for calculating the probability of kill for a single target element due to fragmenting munitions. The computational model is based on the physical properties of munitions and target elements.

2. The fragment effect model consists of four components: fragment patterns, a fragment drag model, a fragment perforation model and a target element model. A fragmentation warhead is characterized by fragment zones (also called fragment sprays and fragment fans), which are modelled as spherical zones, as illustrated in Figure B.2. Fragmentation arena tests can provide experimental data on the warhead fragmentation patterns [4, 26]. A target element is described by a collection of armour segments facing different directions. The simple kill criterion used here states that the target is considered killed if any of its armour segments are sufficently perforated by fragments or damaged by blast. The armour segments are considered independent of each other.

3. Only some basic equations are presented here. For simplicity, only a single kill type is considered. More elaborate target element models with advanced kill rules can be constructed. The terrain features at the target area can also be taken into account using a digital elevation model.

B.2.2 Calculating Fragment and Blast Effects to a Single Target Element

1. This section presents a simple algorithm for calculating the kill probability of a single fragmenting warhead to a single target element. It is assumed that the warhead detonates above the ground. The geometry of the munition/target element interaction is illustrated in Figure B.1.

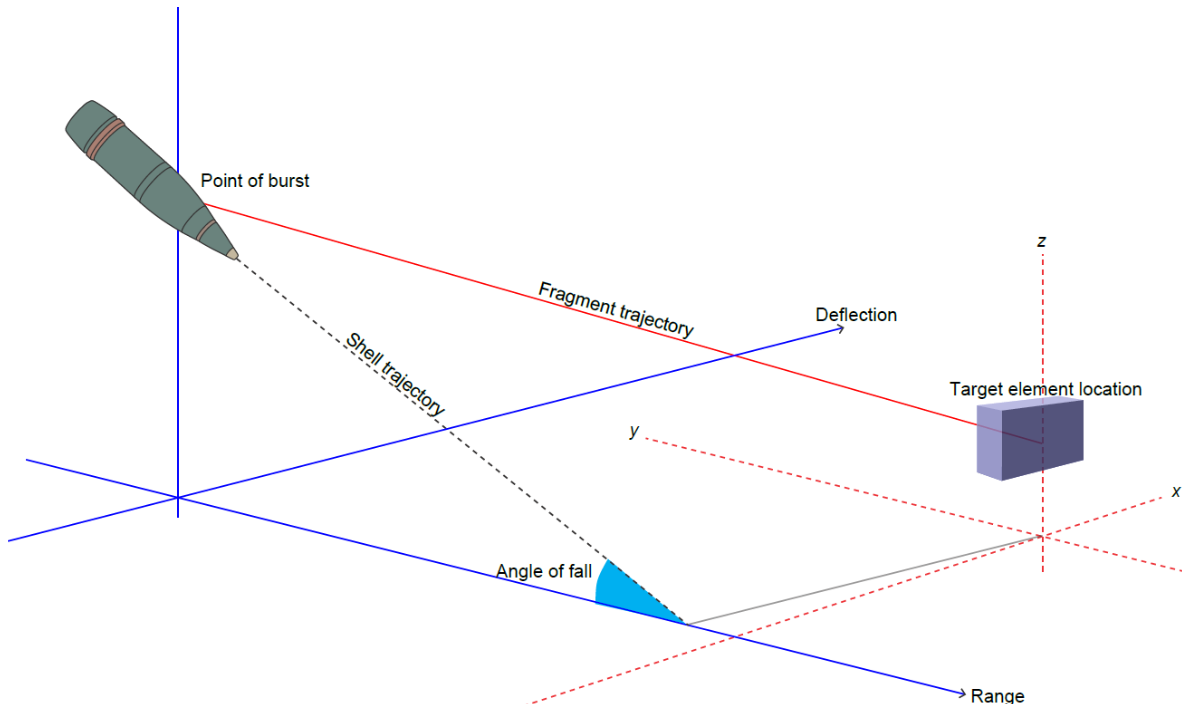

Figure B.1. Terminal ballistics geometry. Figure adapted from [27].

2. The following inputs are needed:
   - Target element location and orientation
   - Munition velocity vector and desired point of burst
   - Munition and target element parameters, see Section B.2.3
   - Optional: digital elevation model of the target area

3. The algorithm for computing the probability of kill is outlined as follows:
   1 Determine the point of burst based on fuze settings and terrain
   2 For all armour segments *j* in target element:
      2.1 Calculate distance from point of burst to segment
      2.2 Calculate blast kill probability *P*(blast kill), see Section B.2.4.5
      2.3 For all fragment zones *i* in the munition:

2.3.1 Calculate dynamic zone angles, Eq. (B-10)
2.3.2 Check that armour segment is within the fragment zone
2.3.3 Calculate projected area of armour segment
2.3.4 Check that armour segment is facing the point of burst
2.3.5 Check for line of sight from point of burst to armour segment
2.3.6 Calculate surface area of fragment zone, Eq. (B-9)
2.3.7 Calculate minimum mass capable of perforation, Eq. (B-15)
2.3.8 Calculate the number of effective fragments, Eq. (B-14)
2.3.9 Calculate fragment kill probability $p_{i,j}$, Eq. (B-11)

3. Calculate overall fragment kill probability from

$$P(\text{fragment kill}) = 1 - \prod_i \prod_j (1 - p_{i,j}) \quad \textbf{(B-1)}$$

4. The kill probability is calculated from

$$P(\text{kill}) = 1 - (1 - P(\text{blast kill}))(1 - P(\text{fragment kill})) \quad \textbf{(B-2)}$$

B.2.3 Input Data

B.2.3.1 Parameters for Fragmenting Munition

1. A fragmenting munition can be described by the following set of parameters.
   - Explosive fill in TNT equivalent mass / alternatively the mass and type of explosive. This is used for determining the blast effect.
   - An arbitrary number of fragment zones (also called fragment sprays), modelled as spherical zones, each having the following information
     - start and end angles with respect to the warhead nose
     - fragment mass distribution (in tabular or functional form)
     - initial fragment speed
     - fragment shape factor or, more generally, a drag model
     - fragment perforation equation (Rilbe, THOR, etc.), chosen based on the shape and material of the fragments

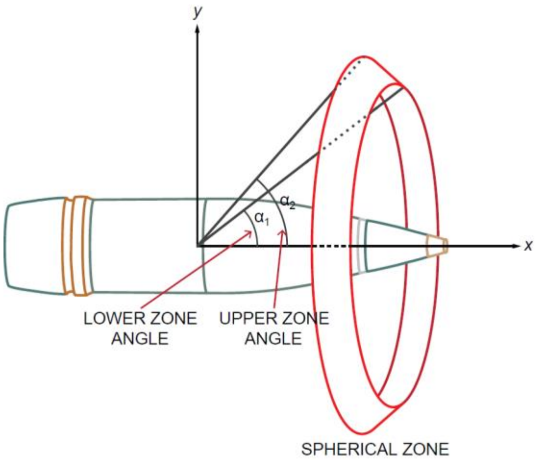

Figure B.2. Fragment zone described as a spherical zone. Figure adapted from [25].

2. The warhead data can be stored in an arbitrary format. One such format is the ZDATA file format [25], in which the fragment mass distribution for each zone is given in tabular form and the fragments have a shape factor used for computing drag.

B.2.3.1.1 Example: 155 mm HE Shell M10

1. The shell has an explosive fill of 6.6 kg TNT [11]. Illustrative, but not necessarily accurate, parameters relating to fragmentation are presented in Table B.1. The total mass of the shell casing is divided over the zones as follows: 15% in the nose zone, 80% in the side zone and 5% in the base zone. The angles of the fragment zones and the fraction of fragments in each zone are based on data for a generic HE shell given in [11].

Table B.1. Fragmentation characteristics of a 155 mm HE shell M107, based on open source data.

Parameter	Nose zone	Side zone	Base zone
Lower zone angle	0°	65°	170°
Upper zone angle	10°	115°	180°
Initial fragment speed at start angle	1030 m/s [18]	1030 m/s [18]	1030 m/s [18]
Initial fragment speed at end angle	1030 m/s [18]	1030 m/s [18]	1030 m/s [18]
Fragment distribution	Mott distribution, 381 fragments with average mass 14.34 g [18]	Mott distribution, 2030 fragments with average mass 14.34 g [18]	Mott distribution, 127 fragments with average mass 14.34 g [18]
Fragment drag model	Irregular fragments	Irregular fragments	Irregular fragments
Fragment perforation model	Rilbe, steel fragments	Rilbe, steel fragments	Rilbe, steel fragments

B.2.3.2 Target Element Parameters

1. The target elements can be represented in three dimensions by a set of armour segments, each having a relative position, a normal vector and an area. Each segment is given a thickness value and material type, e.g. mild steel. Additionally, criteria for blast damage may be added to each segment.

2. This model has the advantage that personnel and vehicles can be handled in a similar manner. It also makes it straightforward to model the effect of protective gear for personnel, as well as different postures.

3. For infantry target elements, one can also use other incapacitation models, such as the Kokinakis and Sperrazza model [17].

B.2.3.2.1 Example: Prone Soldier

1. Dimensions of a prone soldier are presented in Table B.2.

Table B.2. Dimensions of a prone soldier. A fragment capable of perforating 1.5 mm of mild steel is considered sufficient of causing incapacitation. Source of areas: [28].

Aspect	Area [m^2]	Equivalent steel thickness [mm]
Front/Rear	0.08	1.5
Left	0.38	1.5
Right	0.38	1.5
Top	0.61	1.5

B.2.4 Basic Equations

B.2.4.1 Fragment Mass Distributions

1. Let the complementary cumulative distribution function (CCDF) of the fragment mass distribution be deifined as

$$N(M > m) = F_{\mathrm{M,c}}(m) \quad \textbf{(B-3)}$$

2. This is the cumulative number of fragments having a mass greater than a given mass *m*.

3. Below, three fragment mass distributions are presented: a discrete (categorical) distribution, the Mott distribution [22] and the Held distribution [15]. Several other distributions are available as well, see e.g. [13].

B.2.4.1.1 Categorical Distribution

1. A straightforward way of describing a fragment mass distribution in a fragment zone is to divide the fragment masses into ng mass groups. Each group i contains ni fragments with average mass mi.

2. In this case, the CCDF is

$$F_{\mathrm{M,c}}(m) = \sum_{m_i > m} n_i, i = 1,2, \dots, n_{\mathrm{g}} \quad \textbf{(B-4)}$$

B.2.4.1.2 The Mott Distribution

1. The Mott distribution has the following parameters

- N_0 – Total number of fragments
- m_{avg} – Average mass of fragments [kg]

2. The total mass of fragments in the distribution is given by

$$M_0 = N_0 m_{\mathrm{avg}}. \quad \textbf{(B-5)}$$

3. The CCDF of the Mott distribution is given by

$$F_{\mathrm{M,c}}(m;\ N_0, m_{\mathrm{avg}}) = N_0 \mathrm{e}^{-\sqrt{\frac{2m}{m_{\mathrm{avg}}}}} \quad \textbf{(B-6)}$$

B.2.4.1.3 The Held Distribution

1. The Held distribution has the following parameters.

- M_0 – Total mass of fragments in distribution [kg]

- *B* – Scaling factor
- *λ* – Form factor

2. The CCDF of the Held distribution is an implicit function and has to be solved numerically with respect to *N* for a given mass *m*.

$$m = M_0 B\lambda N^{\lambda-1} e^{-BN^{\lambda}} \quad \textbf{(B-7)}$$

B.2.4.2 Fragment Kill Probability

B.2.4.2.1 Fragment Hit Probability

1. Assume an area *A* perpendicular to the fragment path. If the area of the fragment zone A_{zone} is large compared to the area *A*, the probability of fragment hitting the area is

$$p_{\text{hit}} = \frac{A}{A_{\text{zone}}}. \quad \textbf{(B-8)}$$

2. The fragment pattern of a fragmenting munition can be modelled as a spherical zone, defined by an upper and a lower angle. Due to the velocity of the projectile, the angles of the zones will change and the total initial velocity of the fragments will be the resultant of the projectile velocity and the initial velocity in the static case. An illustration of fragment zones for a shell at rest and a munition in motion is shown in Figure B.3. The static angles, when the munition is at rest, are denoted by *α* and the corresponding dynamic zones, when the shell is in motion, by *β*. The area of a spherical zone is

$$A_{\text{zone}} = 2\pi x^2(\cos(\beta_{\text{start}}) - \cos(\beta_{\text{end}})) \quad \textbf{(B-9)}$$

where *x* is the distance and β_{start} and β_{end} are the start angle and end angle of the fragment zone, respectively.

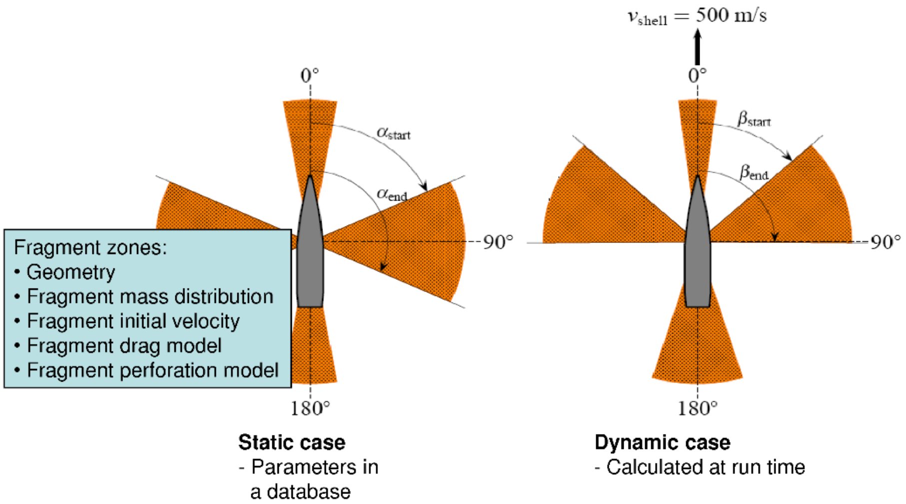

Figure B.3. Fragment zone angles for a shell at rest and in motion. Figure adapted from the presentation by [21].

3. The angles are defined such that 0° ≤ α_{start} ≤ α_{end} ≤ 180°, where 0° is in the direction of the munition's nose.

4. Given the static angle α, the fragment initial speed in the static case and the munition velocity, the dynamic angle can be calculated from

$$\beta = \arccos\left(\frac{v_{shell} + v_{frag}\cos(\alpha)}{v_{tot}}\right) = \arccos\left(\frac{v_{shell} + v_{frag}\cos(\alpha)}{\sqrt{v_{frag}^2 + 2v_{frag}v_{shell}\cos(\alpha) + v_{shell}^2}}\right) \quad \textbf{(B-10)}$$

with:

β	=	dynamic fragment zone angle
α	=	static fragment zone angle
v_{shell}	=	munition speed
v_{frag}	=	fragment speed in the static case
v_{tot}	=	total fragment speed

B.2.4.2.2 A Simple Kill Rule Based on Fragment Perforation

1. The probability of at least k perforating fragment hits is calculated using the binomial distribution

$$P(\text{at least } k \text{ fragment hits}) = 1 - F_{X,\text{Bin}}(k-1; n_{\text{eff}}, p_{\text{hit}}), \quad \textbf{(B-11)}$$

where n_{eff} is the number of effective, i.e., perforating, fragments. $F_{X,Bin}(k; n, p)$ is the cumulative distribution function of the binomial distribution.

2. In the special case of at least one perforating fragment, Eq. (B-11) simplifies to

$$P(\text{at least one perforating fragment hit}) = 1 - \left(1 - \frac{A}{A_{\text{zone}}}\right)^{n_{\text{eff}}} \quad \textbf{(B-12)}$$

$$\approx 1 - \exp\left(-n_{\text{eff}} \frac{A}{A_{\text{zone}}}\right) = 1 - \mathrm{e}^{-\rho_{\text{frag}} A} \quad \textbf{(B-13)}$$

with:

A	=	area of target segment perpendicular to the fragment path [m^2]
A_{zone}	=	area of fragment zone [m^2]
n_{eff}	=	number of effective fragments
ρ_{frag}	=	area density of fragments [1/m^2]

3. Small fragments will lose speed faster than larger ones, which means that large fragments will remain effective over greater distances. Therefore, the smallest effective fragment is to be found. The number of fragments with a mass greater than or equal to a minimum mass m_{min} is then calculated from the fragment mass distribution

$$n_{\text{eff}} = F_{\text{M,c}}(m_{\text{min}}) \quad \textbf{(B-14)}$$

4. The effective fragment is here taken as a fragment capable of perforating an armour plate of a certain thickness e_{min} from a given distance x with an initial speed v_0 and can be computed using the following procedure.

5. The aim is to solve the optimization problem

$$\underset{m}{\text{minimize}}\, m \quad \textbf{(B-15)}$$

subject to the following constraints

$$m > 0 \quad \textbf{(B-16)}$$

$$f_{\text{e}}(m, v_{\text{s}}) \geq e_{\text{min}} \quad \textbf{(B-17)}$$

where $f_e(\cdot)$ is a perforation equation. The striking speed v_s is given by a drag model

$$v_{\text{s}} = v(m;\ x, v_0) \quad \textbf{(B-18)}$$

6. This can be solved as a constrained nonlinear program. It can also be set up as a nonlinear root search problem.

7. Instead of using a perforation model, the fragment lethality can, e.g., be based on its kinetic energy. In that case the effectiveness threshold e_{min} is defined as a minimum kinetic energy and function $f_e(\cdot)$ as the formula for kinetic energy.

B.2.4.2.3 The Kokinakis and Sperrazza Model for Incapacitation

1. This section demonstrates how the Kokinakis and Sperrazza model for incapacitation of personnel from fragments can be incorporated into the model.

2. Kokinakis and Sperrazza presented in 1965 an empirically derived criterion for incapacitation of soldiers from steel fragments and flechettes [17]. The conditional probability that a soldier is incapacitated from a fragment hit is calculated using the following expression

$$P(\text{incapacitation} \mid \text{hit})(m, v_s) = p_{\text{I|H}}(m, v_s) = 1 - \exp\left(-a(mv_s^{3/2} - b)^n\right) \qquad \textbf{(B-19)}$$

with:

m	=	fragment mass [kg]
v_s	=	fragment striking speed [m/s]
a, b, n	=	parameters, see Table B.3

3. The values of parameters a,b,n are associated with a tactical role and a post-wounding time. Tables of parameters are given in [17] for soldiers with no clothes and soldiers wearing American winter uniforms and helmets. The parameters are given for major body parts and for the body as a whole. Depending on the tactical role, the different body parts are given different weightings. The post-wounding time denotes the time between the hit and the incapacitation (see Table B.3).

Table B.3. Parameters in SI units for Kokinakis and Sperrazza equation (B-19) for steel fragments. The parameters are for the entire body. Winter uniform means American winter uniform with helmet and boots. Source: [17].

Role	**Clothing**	***a***	***b***	***n***
Defence ≤ 30 s	Winter uniform	0.15368	0.34239	0.45106
Assault ≤ 30 s	Winter uniform	0.22039	0.33803	0.49570
Assault ≤ 5 min	Winter uniform	0.275420	0.33803	0.48781
Supply ≤ 12 h	Winter uniform	0.34891	0.31622	0.44350
Defence ≤ 30 s	None	0.18934	0.28896	0.41356

4. The probability of hit, p_{hit}, is computed using Eq. (B-8). If the fragment mass distribution is categorical with k fragment size categories, each with N_i fragments with mass m_i, $i \in [1, k]$, the probability of incapacitation is given by

$$P(\text{incapacitation}) = 1 - \prod_{i=1}^{k}\left(1 - p_{\text{hit}} p_{\text{I|H}}(m_i, v_s(m_i))\right)^{N_i} \quad \textbf{(B-20)}$$

5. If the fragment mass distribution is continuous, the probability of incapacitation is given by

$$P(\text{incapacitation}) = 1 - \exp\left(\int_0^{\infty} f_{\text{M}}(m) \ln\left(1 - p_{\text{hit}} p_{\text{I|H}}\right) dm\right) \quad \textbf{(B-21)}$$

where $f_M(m)$ denotes the mass function, whose relationship with the CCDF, $F_{M,c}(m)$, is

$$\mathrm{F}_{\mathrm{M,c}}(m) = \int_m^{\infty} f_M(m) dm \quad \textbf{(B-22)}$$

B.2.4.3 Fragment Drag Models

1. The drag models provide the fragment speed at a distance x, given an initial speed of v_0. They are used for calculating the impact speed of the fragment, when the point of detonation and the position of the target element are known. Two drag models are presented here. The first one has a general form and is applicable to fragments with regular shape. The second one is intended for irregular fragments, formed by natural fragmentation.

B.2.4.3.1 General Drag Model

1. A general drag model can be derived from the drag equation and Newton's second law,

$$v(x) = v_0 \exp\left(-\frac{\rho_a C_d A x}{2m}\right), \quad \textbf{(B-23)}$$

with:

$v(x)$	=	speed at distance x [m/s]
x	=	the distance traversed [m]
v_0	=	the initial speed [m/s]
ρ_a	=	density of air [kg/m^3] ($\rho_a \approx 1.2$ kg/m^3)
C_d	=	drag coefficient [-]
A	=	cross-sectional area of the object [m^2]
m	=	fragment mass [kg]

2. The drag coefficient C_d depends on the shape and orientation of the fragment and on the Mach number M and the Reynolds number Re. The value of the Reynolds number gives an indication about the type of fluid flow around an object. The variation with Reynolds number is usually small within practical regions of interest, and the dependency is therefore ignored. Examples of drag coefficients for cubes and spheres are listed in Table B.4.

Table B.4. Drag coefficients for various shapes. Mach region indicates the Mach values for which the drag coefficient has been defined.

Shape	C_d	Mach Region	Reference
Cube	0.83	$M \le 0.9$	[4]
Cube	1.14	$M > 0.9$	[4]
Sphere	0.49	$M \le 0.9$	[4, 16]
Sphere	0.93	$M > 0.9$	[4, 16]

B.2.4.3.2 Drag Model for Irregular Fragments

1. A fragment shape factor f_k can be introduced,

$$f_k = \frac{A}{m^{2/3}} \quad \textbf{(B-24)}$$

with:

f_k = fragment shape factor [$m^2/(kg)^{2/3}$]
A = cross-sectional area of the object [m^2]
m = fragment mass [kg]

2. Substituting Eq. (B-24) into Eq. (B-23), the following equation is derived

$$v(x) = v_0 \exp\left(-\frac{kx}{\sqrt[3]{m}}\right) \quad \textbf{(B-25)}$$

where:

$$k = \frac{\rho_a C_d A}{2m^{2/3}} = \frac{1}{2}\rho_a C_d f_k \quad \textbf{(B-26)}$$

3. The value of coefficient k is determined experimentally and represents an average for all fragments in the fragment zone. In [10, 3] the value k = 0.004 $(kg)^{1/3}/m$ is given for a steel fragment of some standard shape. The following parameter values for a steel fragment are given in [23], the original source being Swedish FOA:

$$k = \begin{cases} 0.00264, & M \le 0.9 \\ 0.00456, & M > 0.9 \end{cases} \quad \textbf{(B-27)}$$

B.2.4.4 Fragment Perforation Equations

B.2.4.4.1 The Rilbe Formula

1. The Rilbe formula [24] is used to compute the perforation capacity of a fragment with a certain mass and striking speed. The formula is expressed as

$$e = q v_s m^{1/3} \quad \textbf{(B-28)}$$

with:

e = the penetration depth [m]

q = **a parameter that depends on the fragment material, target material and fragment shape [s(kg)$^{-1/3}$], see**

v_S = the striking speed [m/s]

m = the fragment mass [kg]

[24] gives some values for constant *q* for a few combinations of fragment and target material, See

2. .

Table B.5. Rilbe constants for Eq. (B-28). Source [24].

		***q* [s(kg)$^{-1/3}$]**	
Fragment		**Target material**	
Material	**Shape**	**Mild steel (SIS 1311)**	**Dural**
Steel	Soft sphere (HRC 12) Cube Natural fragment	$56 \cdot 10^{-6}$ $42 \cdot 10^{-6}$ $39 \cdot 10^{-6}$	$115 \cdot 10^{-6}$ $90 \cdot 10^{-6}$ $82 \cdot 10^{-6}$ (calculated) / $70 \cdot 10^{-6}$ (experimental)
Tungsten	Small sphere (diam. ≤12 mm) Cube	$72 \cdot 10^{-6}$ $61 \cdot 10^{-6}$	$190 \cdot 10^{-6}$ $150 \cdot 10^{-6}$

B.2.4.4.2 The THOR Equations

1. There is a number of variations of the THOR formula [6, 7, 9, 12]. One can use the formula to estimate the residual speed of the fragment after exiting the target plate or the striking speed necessary to perforate a target plate of a specific thickness. The equations can be simplified by making assumptions about the fragment shape.

2. By setting the residual fragment speed to zero, we obtain the ballistic limit for a general fragment shape

$$v_s = 10^{c_{1,SI}}(eA)^{\alpha_1} m^{\beta_1} (\sec(\theta))^{\gamma_1} \quad \textbf{(B-29)}$$

and for a specific fragment

$$v_s = 10^{c^*_{1,SI}} e^{\alpha_1} m^{\beta^*_1} (\sec(\theta))^{\gamma_1} \quad \textbf{(B-30)}$$

with:

v_s = the fragment striking speed [m/s]

e = the target thickness [m]

A = the average impact area of the fragment [m^2]

m = the mass of the fragment [kg]
θ = the angle between the trajectory of the fragment and the normal to the target material
$c_{1,SI}$, $c_{1,SI}^*$, c_1^*, α_1, β_1, β_1^*, γ_1 = constants determined separately for each material, see Table B.6.

3. Constants for the THOR equations for steel fragments can be found in for example [6, 7, 9].

B.2.4.5 Blast Damage

1. A blast wave generated in air and transmitted through the air is characterized primarily by a peak overpressure and a specific impulse, the latter being the integral of the overpressure over the positive phase time duration.

2. There are diagrams and numerical models available for TNT for determining the peak overpressure and impulse as a function of the distance from the point of detonation. Such diagrams are given e.g. in [2, 3]. For explosives other than TNT, one can convert the mass to a TNT equivalent by multiplying with a scaling factor [8]. As some of the energy released by the detonation goes into fracturing the shell casing, this also needs to be considered, by converting the mass into bare equivalent charge.

3. Threshold values for various levels of damage to personnel and structures from overpressure and impulse can be found in literature, enabling a simple three-dimesional cookie cutter damage function to be used for detonations in free air.

B.2.5 References

[1] Kenttätykistöopas I: Ampumaoppi. Pääesikunnan koulutusosasto, 1990.

[2] Technical manual TM 5-1300: Structures to resist the effects of accidental explosions. Technical Report Army TM 5-1300, Navy NAVFAC P-397, Air Force AFR 88-22, Washington, DC, November 1990.

[3] United facilities criteria (UFC): Structures to resist the effects of accidental explosions. UFC UFC 3-340-02, Washington, D.C., 2008.

[4] US Army Combat Systems Test Activity. GE/UK/US "Static testing of high explosive munitions for obtaining fragment spatial distributions". International Test Operations Procedure, December 1990. ITOP 4-2-183.

[5] Bernt M. Åkesson, Esa Lappi, Ville H. Pettersson, Eric Malmi, Sampo Syrjänen, Marko Vulli, and Kari Stenius. Validating indirect fire models with field experiments. *Journal of Defense Modeling and Simulation: Applications, Methodology, Technology*, 10(4):425–434, October 2013.

[6] Ballistic Analysis Laboratory, The Johns Hopkins University. A comparison of the performance of fragments of four materials impacting on various plates. Technical Report Project Thor Technical Report No. 41, Institute for Cooperative Research, The Johns Hopkins University, Baltimore, MD, May 1959.

[7] Ballistic Analysis Laboratory, The Johns Hopkins University. The resistance of various metallic materials to perforation by steel fragments, empirical relationships for fragment residual velocity and residual weight. Technical Report Project Thor Technical Report No. 47, Institute for Cooperative Research, The Johns Hopkins University, Baltimore, MD, April 1961.

[8] Paul W. Cooper. *Explosives Engineering*. Wiley, New York, 1996.

[9] Michelle Crull and Michael M. Swisdak, Jr. Methodologies for calculating primary fragment characteristics. Technical Paper 16, Revision 2, Department of Defense Explosives Safety Board, Alexandria, VA, October 2005.

[10] Department of the Army, Washington, D.C. *Technical Manual TM 5-855-1: Fundamentals of protective design from conventional weapons*, 1986.

[11] Ove Dullum. Cluster weapons – military utility and alternatives. Technical Report FFI-rapport 2007/02345, Norwegian Defence Research Establishment (FFI), 2008.

[12] Donald O. Dusenberry, editor. *Handbook for Blast Resistant Design of Buildings*. Wiley, 2010.

[13] Predrag Elek and Slobodan Jaramaz. Fragment mass distribution of naturally fragmenting warheads. *FME Transactions*, 37(3):129–135, 2009.

[14] Tapio Heininen. A method to calculate the lethality of fragmenting ammunition. In J. S. Hämäläinen, editor, *Lanchester and Beyond—A Workshop on Operational Analysis Methodology*, volume 11 of *Defence Forces Technical Research Centre Publications*, pages 19–30, Riihimäki, 2006. Defence Forces Technical Research Centre.

[15] Manfred Held. Fragment mass distribution of he projectiles. *Propellants, Explosives and Pyrotechnics*, 15:254–260, 1990.

[16] Bo Janzon. Underlag för verkansberäkningar. H5. Splitters luftmotstånd. Technical Report FOA 2 A 2539-44, Försvarets forskningsanstalt, Stockholm, 1971.

[17] William Kokinakis and Joseph Sperrazza. Criteria for incapacitating soldiers with fragments and flechettes. BRL Report BRL R 1269, Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland, 1965.

[18] Theodor Krauthammer. *Modern Protective Structures*. CRC Press, 2008.

[19] Esa Lappi. *Computational Methods for Tactical Simulations*. Doctor of Military Science Thesis, National Defence University, Finland, 2012.

[20] Esa Lappi, Olli Pottonen, Sami Mäki, Kosti Jokinen, Olli-Pentti Saira, Bernt M. Åkesson, and Marko Vulli. Simulating indirect fire—a numerical model and validation through field tests. In *2nd Nordic Military Analysis Symposium*, Stockholm, Sweden, 2008.

[21] Esa Lappi, Mikko Sysikaski, Bernt Åkesson, and Uğur Z. Yildirim. Effects of terrain in computational methods for indirect fire. In C. Laroque, J. Himmelspach, R. Pasupathy, O. Rose, and A. M. Uhrmacher, editors, *Proceedings of the 2012 Winter Simulation Conference*, 2012.

[22] N. F. Mott. Fragmentation of H.E. shells: A theoretical formula for the distribution of weights of fragments. Army Operations Group Research Memo No. 24 AC3642, Ministry of Supply, United Kingdom, March 1943.

[23] Risto Noopila. Kranaattien sirpaloitumiselle asetettavat vaatimukset [Requirements set on the splintering of shells]. In P. Multanen, R. J. Setälä, S. Ahto, P. Hakala, and R. Lahtiperä, editors, *Tiede ja Ase – Suomen Sotatieteellisen Seuran Vuosijulkaisu N:o 42 [Science and Weapons – The Annual of the Finnish Society of Military Sciences Issue 42]*, pages 144–180. Suomen Sotatieteellinen Seura (The Finnish Society of Military Sciences), Helsinki, 1984.

[24] Ulf Rilbe. Splitters genomslagsförmåga—Sammanställning av numeriska värden grundade på försök och beräkningar [Perforation capability of fragments—Compilation of numerical values based on experiments and calculations, in Swedish]. Technical Report FOA 2 A 2525-44, Försvarets forskningsanstalt, Stockholm, 1970.

[25] Robert J. Yager. Reading, writing, and analyzing ZDATA fiels using C++. Technical Report ARL-TN-541, Army Research Laboratory, Aberdeen Proving Ground, MD, October 2013.

[26] STANAG 4589 Ed 1, Static testing of high-explosive munitions for obtaining fragment spatial distribution.

[27] Morris Driels. Weaponeering: Conventional Weapon System Effctiveness, AIAA Education Series, American Institute of Aeronautics and Astronautics, 2004.

[28] Tapio Saarelainen. Taistelija 2020 – tulevaisuuden kärkitaistelija, Maasotakoulu, Lappeenranta, 2007.

B.3. THE CARLETON DAMAGE FUNCTION FOR UNITARY FRAGMENTING MUNITIONS (MEDIUM FIDELITY)

1. The Carleton Damage Function for determining the probability of kill (P_k) for a single target element due to a fragmenting munition is specified in equation B-31, B-32, and B-33.

$$P_k(u,v,r,d) = D_0 \cdot e^{-D_0\left(\frac{(u-r)^2}{r_1^2}+\frac{(v-d)^2}{r_2^2}\right)} \quad \textbf{(B-31)}$$

where:

$$r_1^2 = \frac{A_L \cdot Q}{\pi} \quad \textbf{(B-32)}$$

$$r_2^2 = \frac{A_L}{Q \cdot \pi} \quad \textbf{(B-33)}$$

with:

P_k	=	Probability of kill for a single target element
D_0	=	Shaping factor
(u, v)	=	Range and deflection coordinates of a target element (relative to the gun-to-target line)
(r, d)	=	Range and deflection coordinates of the munitions functioning point (relative to the gun-to-target line)
A_L	=	Lethal area of the munition against a target element for a specific Angle of Fall (ω). If ω is changed, the corresponding A_L needs to be substituted into Equation B-32 and B-33
Q	=	General parameter to fit the Carleton damage function to national experimental data. A typical value of Q = (1 – 0.8*cos ω)
r_1,r_2	=	lethal radii values calculated using the ratio of deflection to range and the projectile angle of fall

Assumptions:

1. Intended for use with Gaussian delivery error.
2. Delivery error should be “large” compared to r_1,r_2 (i.e., not intended for guided/precision delivery).

B.4. THE CIRCULAR COOKIE-CUTTER DAMAGE FUNCTION FOR UNITARY FRAGMENTING MUNITIONS (LOW FIDELITY)

1. The Circular Cookie-Cutter Damage Function for determining the probability of kill (Pk) for a single target element due to a fragmenting munition is given in equation B-34. Note that the damage function Pk is multiplied by the munition fuze reliability to yield the total Pk for a single fragmenting munition.

$$P_k(u,v,r,d) = \begin{cases} C & \text{if } (u-r)^2 + (v-d)^2 \le \dfrac{A_L}{C \cdot \pi} \\ 0 & \text{if } (u-r)^2 + (v-d)^2 > \dfrac{A_L}{C \cdot \pi} \end{cases} \qquad \textbf{(B-34)}$$

with:

P_k	=	Probability of kill for a single target element
(u, v)	=	Coordinates of a target element
(r, d)	=	Coordinates of the munition's functioning point
A_L	=	Lethal area of the munition against a target element
C	=	Constant value provided to the model ($0 < C \le 1$)

Assumptions:

1. P_k is constant over the whole lethal fragmentation area of a munition.

B.5. THE COOKIE-CUTTER DAMAGE FUNCTION FOR A PATTERN OF BOMBLET MUNITIONS (LOW FIDELITY)

1. The Cookie-Cutter Damage Function for determining the probability of kill (Pk) for a single target element located within the pattern area of bomblets (the 'donut' form) is specified below and illustrated in Figure B.4. Note the bomblet dispense function will be determined for each carrier round using the carrier reliability RC. Only bomblet reliability is taken into account in the formula below. To yield the total Pk for a single pattern of bomblets, the damage function Pk is multiplied by the reliability of the carrier (RC).

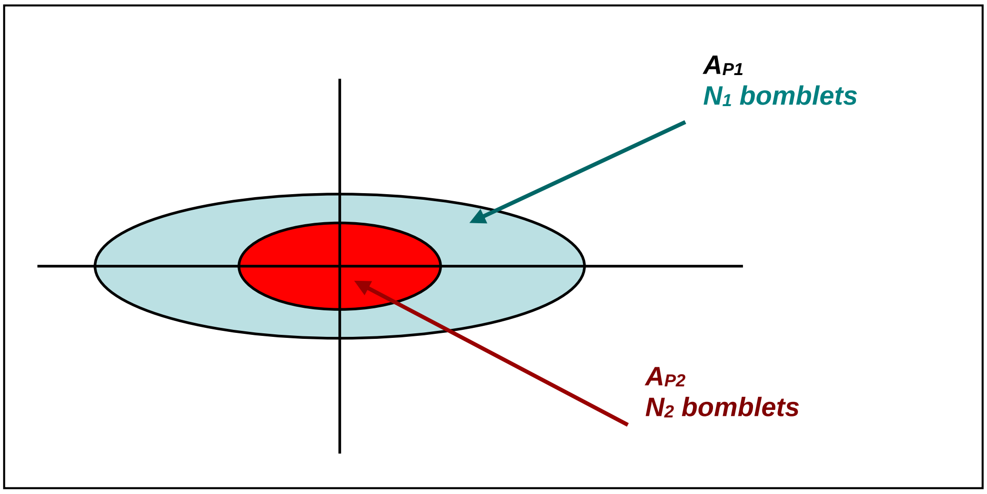

Figure B.4. Plot of cookie-cutter damage function.

$$N = N_1 + N_2 \quad \textbf{(B-35)}$$

$$P_{k1} = \left(1 - \exp\left(\frac{-N_1 \cdot A_L \cdot R_S}{A_{P1}}\right)\right) \quad \textbf{(B-36)}$$

$$P_{k2} = \left(1 - \exp\left(\frac{-N_2 \cdot A_L \cdot R_S}{A_{P2}}\right)\right) \quad \textbf{(B-37)}$$

with:

P_{k1}	=	Probability of kill for a single target element located in the pattern area A_{P1}
P_{k2}	=	Probability of kill for a single target element located in the pattern area A_{P2}
N_1, N_2, N	=	Number of bomblets in A_{P1}, A_{P2}, and total
A_L	=	Lethal area of a single bomblet against the given target element
R_S	=	Reliability of a single bomblet
A_{P1}, A_{P2}	=	Pattern areas over which the N_1 and N_2 bomblets are dispersed (Determination of the pattern areas, A_{P1} and A_{P2}, depends on the shape of the pattern)

Assumptions:

1. Method is valid for both rectangular and elliptical patterns
2. Individual bomblet damage effects are independent

3. Individual bomblet damage effects are enclosed within the bomblet pattern
4. Bomblet distribution within pattern is statistical uniform
5. Statistical guidance suggests n > 50; R$*\frac{AL}{AP}$ should be relatively small: (n$*$R$*\frac{AL}{AP}$) < 5

B.6. MODELLING INDIVIDUAL BOMBLET MUNITIONS

1. Bomblet impact points may be modelled individually. If this is done the damage assessment for each bomblet may be accomplished using any of the damage functions for unitary fragmenting munitions or a damage matrix.

B.7. DAMAGE MATRIX METHODOLOGY (HIGH FIDELITY)

1. This section describes the steps for using damage matrices to calculate the P_k of a target element, and includes examples of the definitions of a damage matrix and target area.

2. A damage matrix is specific to a target element-munition pairing with the target element having a specific state and/or posture and the munition having a specific height of burst and angle of fall. A damage matrix of n x m cells is defined by the following (see Figure B.5):

- A set of n x m cells each containing a P_k value
- A set of n+1 gridline coordinates in range
- A set of m+1 gridline coordinates in deflection
- Functioning point in range and deflection

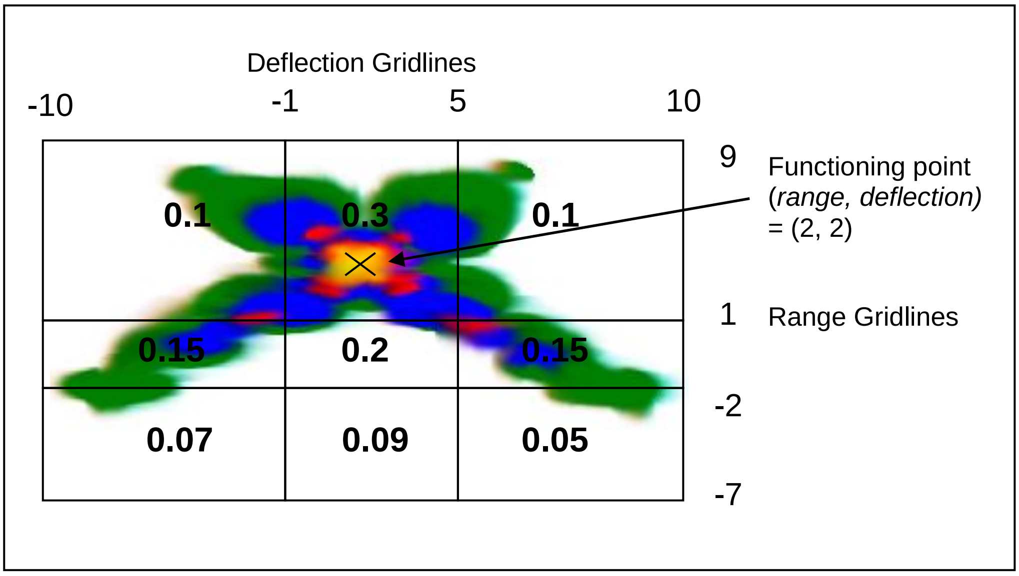

Figure B.5. Damage Matrix shown with sample values.

3. A target area is defined and a functioning point is calculated (see Figure B.6):

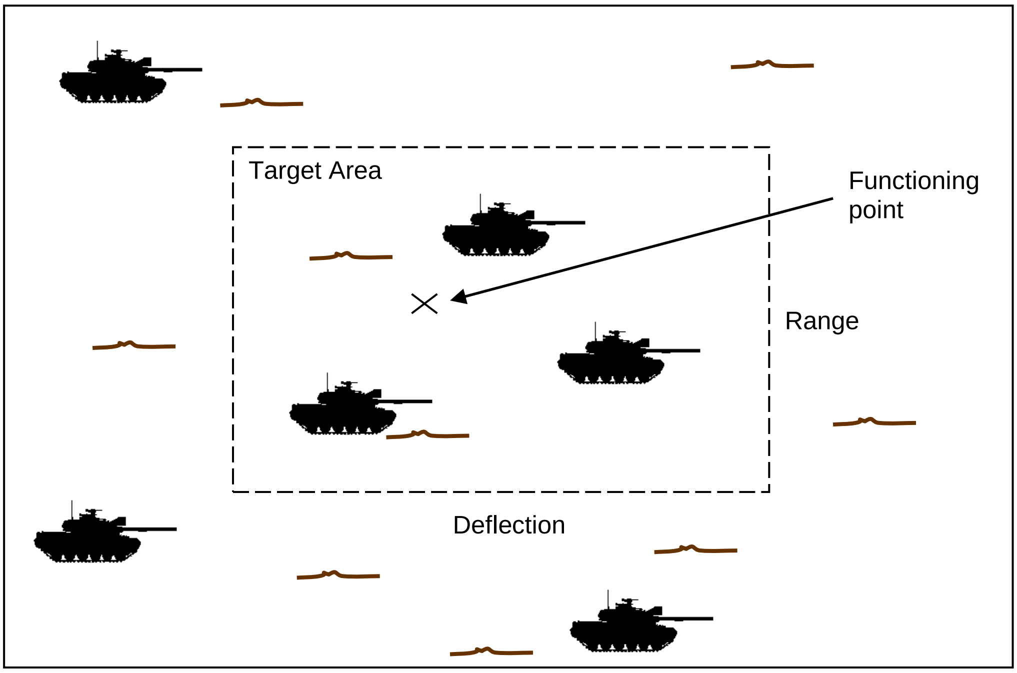

Figure B.6. Target Area example shown from a real world situation.

4. With a damage matrix and target area defined, the P_k for a specific target element within the target area can be calculated. This is accomplished by collocating the damage matrix functioning point and the target area functioning point as shown in Figure B.7. If the target element is located within the damage matrix, the P_k of the corresponding cell is selected.

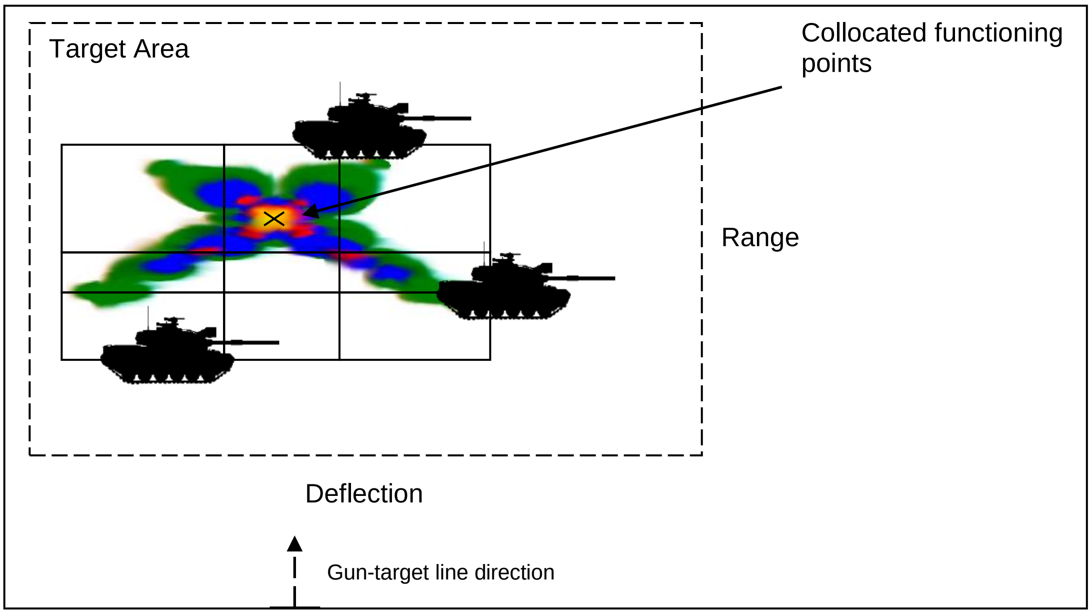

Figure B.7. Target Area overlaid with Damage Matrix.

B.8. MODELLING SENSOR FUZED MUNITIONS

1. Sensor Fuzed Munitions are depicted in Figure B.8.

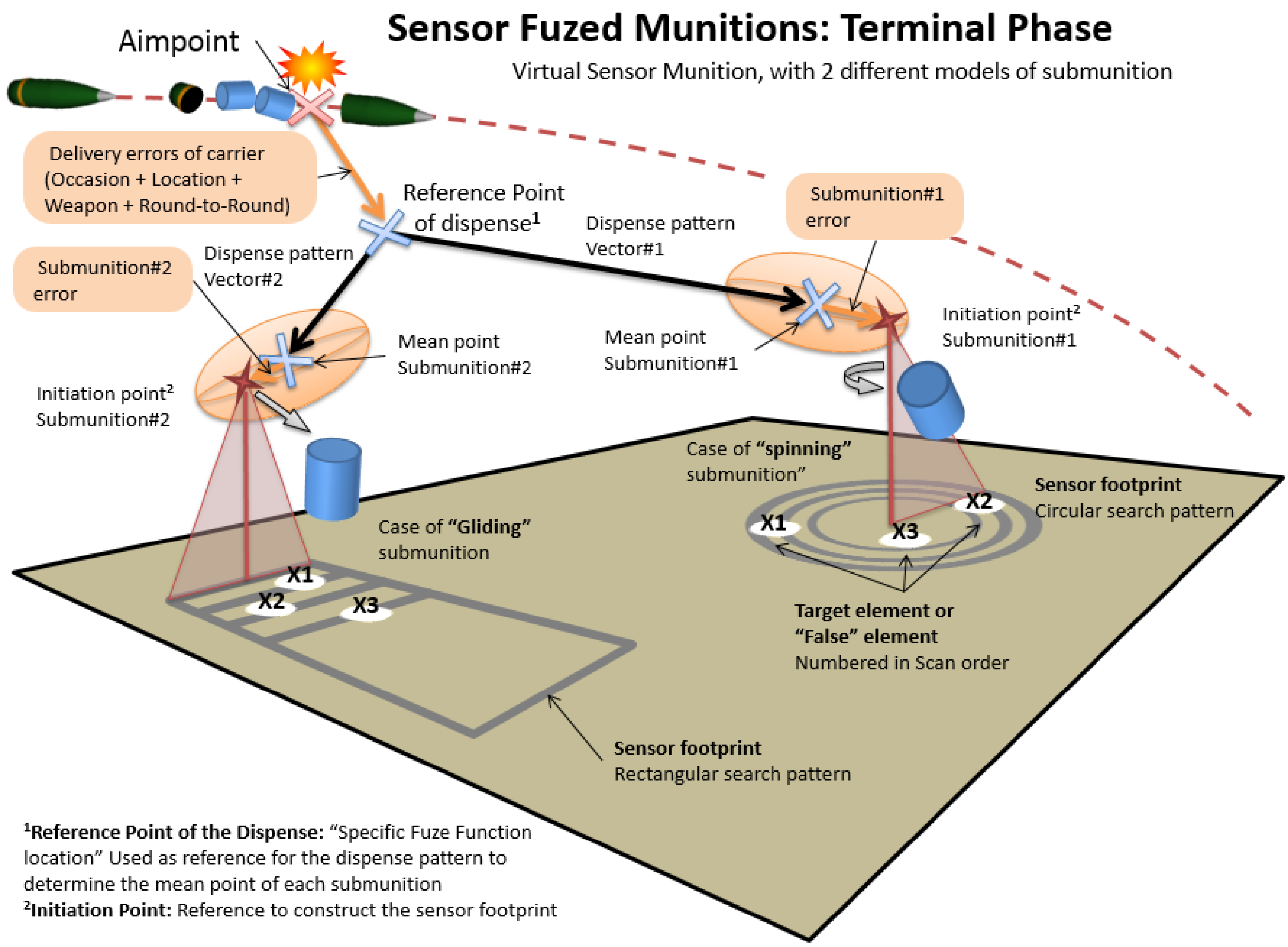

Figure B.8. Sensor Fuzed Munitions.

2. Reliability for munitions is taken into account for calculating effectiveness; however, the reliability of the carrier Rc is accounted for separately from calculation of probability of kill and the term is not included inside of the damage functions themselves. Only the submunition reliability is taken into account in the formula.

B.8.1 Effectiveness Calculation

1. For a single target element and a single submunition, the effectiveness of Sensor Fuzed Munitions can be modelled using the Probability of Kill for a Single Submunition (P_{KSS}) formula:

$$P_{KSS} = R_s \bullet P_{Engage} \bullet P_{Hit|Engage} \bullet P_{Kill|Hit} \quad \textbf{(B-38)}$$

with:

R_S	=	Reliability of the submunition; probability the submunition itself will properly dispense and function
P_{Engage}	=	Probability of Engagement; probability that the target element in a sensor footprint is detected and selected for engagement by a submunition within the presence of a false target. Selection is based on specific submunition selection logic.
$P_{Hit\|Engage}$	=	Probability of Hit given Engagement; probability the submunition will hit a target element selected for engagement.
$P_{Kill\|Hit}$	=	Probability of Kill given a Hit; probability the submunition will kill a target element given it hit the target element.

If the following condition hold:

- The target element is in the sensors footprint.

B.8.2 False Target Element Generation

1. By default, false target elements are uniformly distributed over the battlefield with a density defined as a property of each individual sensor and environment. The false target element density, D_{FT}, is specified in number of false target elements per unit area. If the sensor footprint, A_S, of a munition is small compared to the entire battlefield, then the number of false target elements occurring in the sensor footprint can be shown to be distributed as a Poisson process with an expected value of $D_{FT}*A_S$. The probability that x false target elements occur in the sensor footprint is given by:

$$P(X = x) = e^{(-D_{FT}*A_S)}(D_{FT} * A_S)^x / x! \quad \textbf{(B-39)}$$

2. Using Equation (B-39), V(x) can be computed which consists of the probabilities that x or fewer false target elements will occur in the sensor footprints. The number of false target elements can then be instantiated stochastically by generating a uniform random number, h, and determining the value of x such that

$$V(x-1) < h \le V(x) \quad \textbf{(B-40)}$$

3. Once the number of false target elements has been determined, locations for each are generated stochastically using a uniform distribution.

B.8.3 Sensor Fused Munition Simulation

a. Carrier reliability will be determined for each round using R_C and a stochastic process. If the carrier is determined to be unreliable, no further processing is done for the round.

b. Submunition reliability will be determined for each submunition using R_S and a stochastic process. If a submunition is found to be unreliable, no further processing will be done for that submunition.

c. The initiation points of the submunitions will be determined using the reference point of the dispense, the provided dispense pattern parameters and a stochastic process to simulate delivery errors.

d. Submunitions sensor footprints will be constructed on the ground based on where the submunitions are dispensed and the sensor footprint parameters.

e. The model will determine which target elements are within the sensor footprint. Also, the model will use the false target generation model (see section B.8.2) to determine the number and location of false target elements in the seeker footprint. All target elements (including false target elements) in the footprint are considered to be encountered.

f. All encountered target elements (including false target elements) are sorted by what is referred to as a scan order. Scan ordering is used so that target elements can be considered in an order that would approximate the order in which they would have been scanned by an actual sensor. Scan order is a function of seeker footprint shape type.

g. Target elements (including false target elements) are processed in order of their scan order to determine if an engagement occurs.

1. Detection is determined by using Probability of Detection (P_{Det}, the probability a single submunition will detect a target element in its seeker footprint) (or the Probability of False/Unintended Target Element Detection (P_{FTD}) for false target elements) and a stochastic process.

2. If detection occurs engagement is determined by applying the engagement logic.

h. If a false target element is engaged no further processing is required. If a real target element is engaged the model will determine if the engaged target element is hit by using $P_{Hit|Engage}$ and a stochastic process. If the target element is hit the model will determine if the target element is killed by using $P_{Kill|Hit}$ and a stochastic process.

ANNEX C MODELLING CALCULATION OF DELIVERY ERRORS

1. Mean Point of Impact (MPI) and Round-to-Round errors have a tendency to increase as the distant between the weapon and the impact point increases. This increase has two components, range and deflection, and can be modelled linearly if an estimation is needed because no real data is available.

2. The equation used to estimate delivery errors is:

$$E = A_0 + R*A_1 \qquad \textbf{(C-1)}$$

where:

E	=	The round to round error in the deflection direction, the round to round error in the range direction, the MPI error in the deflection direction or the MPI error in the range direction.
A_0	=	The portion of the error that is constant.
A_1	=	The coefficient that changes the error as range increases.
R	=	The distance from the gun to the aimpoint.

3. The coefficients, A_0 and A_1, are based on a regression of real data available. These coefficients will change based on:

   1. Whether it is an MPI or Round to Round error that is being calculated
   2. The type of weapon that fires the projectile (howitzer, mortar, etc.)
   3. The method of fire (Predicted, Adjusted, etc.)
   4. The angle of fire (High or Low)
   5. The met staleness (1/2 hour, 2 hours, 4 hours, etc.).

4. The following graph (Figure C.1) shows the type of data collected that is used to determine the coefficients.

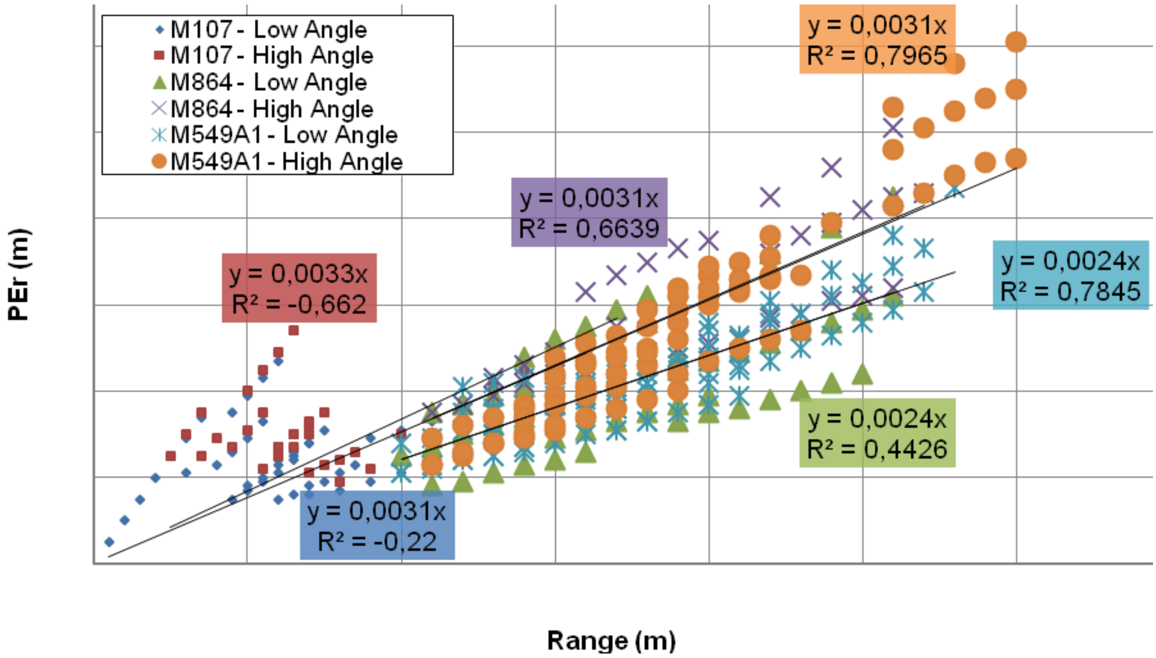

Figure C.1. Probable Error in Range vs. Range for Select 155mm Artillery.

ANNEX D TARGET BEHAVIOUR

D.1. TARGET BEHAVIOUR MODEL

1. During the simulation of a mission, it is not unusual to want to model changes to a target. These changes for personnel are usually called posture sequencing, while changes to materiel are called target hardening.

2. To model this, the sequence of behaviour must be defined. For this methodology, a table containing the changes of the target during a mission is used. The table will consist of the initial target posture followed by a sequence relating the postures and fraction of the surviving target elements as a function of time. This will allow the simulation to determine what state the target is in at a given time. The percentages are applied to the surviving target elements at the time. At the given times (T_1, T_2 etc.) the posture state changes instantaneously.

3. Table D.1 is an example only of how this may be implemented:

Table D.1. Example of how changes of the target during a mission may be implemented.

Time (seconds)	Fraction in Posture 1	Fraction in Posture 2	Fraction in Posture 3	Fraction in Posture 4
Initial State	1.0	(N/A)	(N/A)	(N/A)
T_1	0.5	0.5		
T_2		0.25	0.25	0.5
…	…	…	…	…
T_n			0.33	0.67

4. The model assumes all target elements are at an initial posture. At a given time, the number of surviving elements will be used to determine the number of target elements in each posture for the particular time of the event. A surviving target element is one that has not been killed. This can be expressed as:

$$E_{P1}(t) = E_S(t) * F_{P1}(t).$$
$$E_{P2}(t) = E_S(t) * F_{P2}(t)$$
…

$E_{PN}(t) = E_S(t) * F_{PN}(t)$,

Such that: $E_{P1}(t) + E_{P2}(t) + \ldots + E_{PN}(t) = E_S(t)$ **(D-1)**

Where: $E_{PN}(t)$ = The number of target elements at posture Pn at time T,
$E_S(t)$ = The number of target elements that have survived at time T,
$F_{PN}(t)$ = The fraction part of the target that is at posture Pn at time T.

Notes:

1. Rounding will occur in the calculation of $E_{PN}(t)$ since the calculation in most case will result in a fractional number of target elements at the posture. However, the summation of the $E_{PN}(t)$ targets at a given time must sum to $E_S(t)$ to ensure target elements across all postures sum to the surviving target elements.

2. Since target behaviour deals with postures for the surviving target elements, it is possible that a target element may go from a more hardened posture to a more vulnerable posture.

D.2. EXAMPLE OF TARGET BEHAVIOUR

1. Table D.2 shows the behaviour for a target.

Table D.2. Behaviour for a target.

Time (seconds)	Fraction in Posture 1	Fraction in Posture 2	Fraction in Posture 3	Fraction in Posture 4
Initial State	1.0	(N/A)	(N/A)	(N/A)
10		0.5	0.5	
20		0.25	0.5	0.25
30				1.0

2. Using the above behaviour, if the target contains 100 elements and a round is fired every 10 seconds, the behaviour of the target is determined as follows. When the first round hits, all target elements are at Posture 1. If 30 elements are killed with that first round, the 70 surviving target elements are distributed at the postures using the 10 second state. This means that 35 target elements are at Posture 2 and 35 are at Posture 3.

3. When the second round arrives, 25 target elements are killed. This means that 45 target elements remain. These target elements are distributed using the 20 second state which means 11 target elements are in Posture 2, 23 target elements are in Posture 3 and 11 target elements are in Posture 4.

4. Finally the third round arrives and 15 target elements are killed. This means that 30 target elements are left. We then move to the 30 second and final target element state which is that all remaining target elements are in Posture 4. For the rest of the simulation all target elements are modelled in this state.

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ANNEX E AIMPOINT PLACEMENT

1. This section includes a limited selection of possible aimpoint placement. Other aimpoint placements, based on national tactics, techniques and procedures (TTP) are not included.

E.1. RECTANGULAR SHEAF

1. A Rectangular Sheaf is an aimpoint area in the shape of a rectangle. The aimpoints are distributed over this rectangular area. The distribution of the aimpoints over the rectangular area differs within the national TTP.

E.2. LINEAR SHEAF

1. A Linear Sheaf is a special case of a rectangular sheaf, when one dimension of the rectangular aimpoint area is a lot smaller than the other dimension. On a linear sheaf, aimpoints are distributed on a single line, parallel on the longest dimension. The distribution of the aimpoints over this single line differs within the national TTP.

E.3. CIRCULAR SHEAF

1. Circular Sheaf aiming will place aimpoints in a circular aimpoint area. The distribution of the aimpoints over the circular area differs within the national TTP.

E.4. CONVERGED SHEAF

1. The centre of the aimpoint area is used as the aimpoint for all weapons.

E.5. PARALLEL SHEAF

1. Parallel Sheaf aiming places aimpoints for one firing unit in the aimpoint area relative to the same location as they are being fired from, referenced by mapping the centre point of the firing location to the centre of the aimpoint area.

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ANNEX F MATHEMATICAL MODEL OF DELIVERY ACCURACY

F.1. DELIVERY ACCURACY IN STANAG 4635

1. STANAG 4635 will be used to determine the delivery accuracy taking into account of:

- Target Location Error, modelled using a bivariate, normal distribution;
- Meteorological error, assuming that the platforms are co-located, modelled using a bivariate, normal distribution;
- Firing unit-to-firing unit, modelled using a bivariate, normal distribution with axes aligned with the range and deflection axes for the associated weapon;
- Weapon-to-weapon errors modelled using a bivariate, normal distribution;
- Carrier shell dispense point, modelled using a bivariate, normal distribution, with axes aligned with the range and deflection axes for the associated weapon. Height of burst errors around the dispense point will also be modelled using a bivariate, normal distribution[3];
- Round-to-round errors modelled using a bivariate, normal distribution with axes aligned with the range and deflection axes for the associated weapon;
- Submunition-to-submunition errors modelled using a normal distribution, whose parameters can be specified (bivariate, normal distribution, annulus etc).The mean value will be determined using the parameters defined in STANAG 4355.

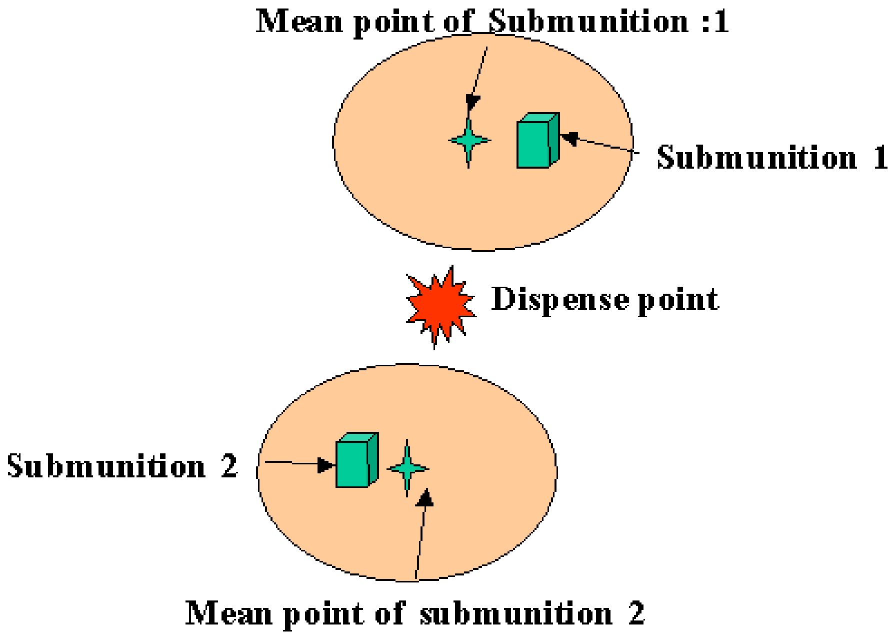

Figure F.1. Submunition-to-Submunition Error.

[3] Note that currently the height of burst error for proximity fuzed fragmenting munitions is not taken into account of in effectiveness models.

F.2. SPATIAL CORRELATION OF MET ERRORS

1. Met errors should be identical for firing units that are in proximity to each other and that are firing the same Munitions at similar ballistic solutions (charge and QE).

2. The following model provides the generation of correlated random deviates for the application of Met errors within a stochastic simulation, to take account of firing units that are not co-located and when firing the same munitions at different ballistic solutions. When firing different munitions, the Met errors are no longer correlated and this method cannot be used.

F.2.1 Generation of Independent Random Numbers

F.2.1.1 Normal Random Numbers

1. Independent normally distributed random numbers will be generated using the Ziggarut method.

F.2.1.2 Uniform Random Numbers

1. Independent uniformly distributed random numbers will be generated using the Mersenne Twister method. The variant of the algorithm known as MT19937 will be used with a 32 bit word length.

F.2.2 Generation of Correlated Normally Distributed Random Numbers

1. Normally distributed random variables with expectation 0 and covariance matrix M can be generated by the following method:

- Find the eigenvalues λ_i and the matrix of eigenvectors **B** of the co-variance matrix **M**.
- Build the matrix **C** = **B S**, wherein:

$$\mathbf{S}=\begin{pmatrix} \sqrt{\lambda_1} & 0 & \cdot & 0 \\ 0 & \sqrt{\lambda_2} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \sqrt{\lambda_n} \end{pmatrix} \qquad \textbf{(F-1)}$$

- Draw n independent normally distributed random numbers z_i (i = 1,…, n) with expectation 0 and standard deviations 1.
- Interpret the z_i as the components of a vector **z**.
- Build the vector **x** = **C z**. The components x_i constitutes the dependent random variates.

ANNEX G DAMAGE THRESHOLD CONTOURS

1. The determination of a Damage Threshold Contour (DTC) for indirect fire missions is accomplished by determining a series of threshold damage distances. These threshold distances are defined as distances beyond which personnel need to be located to maintain a probability of damage below an acceptable level of risk.

2. "Ghost" Target Elements (GTE) are placed along radii projecting from the Target Centre. The "Ghost" Target Element is an artificial target element that is added to the engagement for the sole purpose of computing the DTC. The GTEs are assigned a damage function/matrix consistent with the vulnerability of the subject of the DTC. Figure G.1 shows an example of the GTEs using 8 radii.

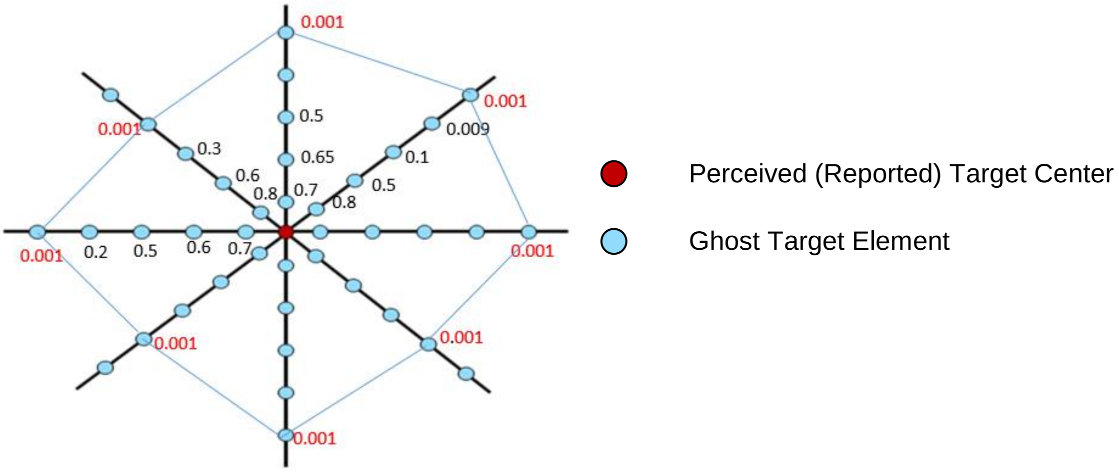

Figure G.1. Example of "Ghost" Target Elements.

3. Using a stochastic approach, the damage level is determined for each GTE. The radials are searched from the farthest ghost point toward the target centre until the first ghost element exhibiting a probability of damage above the threshold value is identified. The location of the previous GTE specifies the contour point along this radial. This is repeated for each radial until the contour is complete.

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ANNEX H LIST OF SYMBOLS

Symbol	Definition	Units
a	*Parameter of Kokinakis-Sperrazza model (B-19)*	
α_{start}	*Upper static fragment zone angle*	°
α_{end}	*Lower static fragment zone angle*	°
α_1	*Constant in THOR equations (B-29 and B-30)*	*none*
A	*Area perpendicular to fragment path*	m^2
A_L	*Lethal Area for given munition(s) against given target element at given angle of fall in a given terrain environment*	m^2
A_{P1}	*Pattern area over which* N_1 *bomblets are dispersed*	m^2
A_{P2}	*Pattern area over which* N_2 *bomblets are dispersed*	m^2
A_S	*Seeker footprint*	m^2
A_{zone}	*Area of fragment zone at a given distance*	m^2
b	*Parameter of Kokinakis-Sperrazza model (B-19)*	*none*
β_1	*Constant in THOR equations (B-29 and B-30)*	*none*
β_1^*	*Constant in THOR equations (B-29 and B-30)*	*none*
β_{start}	*Upper dynamic fragment zone angle*	°
β_{end}	*Lower dynamic fragment zone angle*	°
B	*Scaling factor in Held distribution (B-7)*	*none*
$\mathbf{B}$	*Eigenvector in eigenvalue solution*	*none*
$c_{1,SI}$	*Constant in THOR equations (B-29 and B-30)*	*none*
$c_{1,SI}^*$	*Constant in THOR equations (B-29 and B-30)*	*none*
c_1^*	*Constant in THOR equations (B-29 and B-30)*	*none*
γ_1	*Constant in THOR equations (B-29 and B-30)*	*none*
C	*Constant used as input to the cookie-cutter damage function*	*none*
$\mathbf{C}$	*Matrix used in eigenvalue solution*	*none*
C_d	*Drag coefficient*	*none*
D_0	*Shaping factor of the Carleton damage function*	*none*
d	*Deflection coordinates relative to the gun-target line of the munitions functioning point*	*m*
D_{FT}	*False target element density*	*none*
e	*Thickness of armour segment in perforation models*	*m*
e_{min}	*Minimum armour thickness*	*m*
f_k	*Fragment shape factor*	$m^2/(kg)^{2/3}$
θ	*Angle between fragment trajectory and the normal to the target material in the THOR equations (B-29 and B-30)*	°
h	*Uniform random number (False target element generation)*	*none*
k	*Coefficient for drag model for irregular fragments*	$(kg)^{1/3}/m$
λ	*Eigenvalue, form factor in Held distribution (B-7)*	*none*
m	*Fragment mass*	*kg*
m_{avg}	*Average mass of fragments in Mott distribution*	*kg*
m_{min}	*Minimum effective fragment mass*	*kg*

M	*Covariance matrix used in eigenvalue solution*	*none*
M_0	*Total mass of fragments in Held distribution (B-7)*	*kg*
MET	*Meteorological*	*none*
n	*Number of aimpoints,* *parameter of Kokinakis-Sperrazza model (B-19)*	*none*
n_{eff}	*Number of effective fragments*	*none*
N	*Number of bomblets, number of fragments (B-7)*	*none*
N_0	*Total number of fragments in Mott distribution*	*none*
N_1	*Number of bomblets for pattern area 1*	*none*
N_2	*Number of bomblets for pattern area 2*	*none*
P_{Det}	*Probability of Detection*	*none*
P_{Engage}	*Probability of engagement*	*none*
P_{FTD}	*Probability of false/unintended target element detection*	*none*
p_{hit}	*Probability of hit for fragments*	*none*
$P_{Hit\|Engage}$	*Probability of a hit given an engagement*	*none*
$p_{\mathrm{I\|H}}$	*Conditional probability of incapacitation given a hit*	*none*
$p_{i,,j}$	*Probability of incapacitation by fragments from* *fragment zone i to armour segment j*	*none*
P_k	*Probability of kill for a single target element*	*none*
P_{k1}	*Probability of kill for a single target element located in pattern area* A_{P1}	*none*
P_{k2}	*Probability of kill for a single target element located in pattern area* A_{P2}	*none*
$P_{Kill\|Hit}$	*Probability of kill, given a hit*	*none*
P_{KSS}	*Probability of kill for a single submunition*	*none*
P(fragment kill)	*Probability of kill by fragments*	*none*
P(blast kill)	*Probability of kill by blast*	*none*
P(kill)	*Probability of kill for a single target element*	*none*
P(incapacitation)	*Probability of incapacitation*	*none*
P(incapacitation | hit)	*Conditional probability of incapacitation given a hit*	*none*
q	*Coefficient for the Rilbe formula*	$s(kg)^{-1/3}$
Q	*General parameter to fit the Carleton damage function to experimental data*	*none*
R_C	*Reliability of the carrier munition*	*none*
R_S	*Reliability of a single bomblet*	*none*
r	*Range coordinates relative to the gun-target line of the munitions functioning point*	*m*
r_1	*Parameter used within the Carleton damage function*	*m*
r_2	*Parameter used within the Carleton damage function*	*m*
ρ_{a}	*Density of air*	kg/m^3
ρ_{frag}	*Areal density of fragments*	$1/m^2$
S	*x,y plane used in calculation of lethal area*	*none*
TLE	*Target Location Error*	*m*

u	*Range coordinates relative to the gun-target line of the target element*	*m*
v	*Deflection coordinates relative to the gun-target line of the target element*	*m*
v(x)	*Speed at distance x*	*m/s*
v_0	*Initial speed of fragments*	*m/s*
v_{frag}	*Fragment speed in the static case*	*m/s*
v_s	*Fragment striking speed*	*m/s*
v_{shell}	*Munition speed*	*m/s*
v_{tot}	*Total fragment speed*	*m/s*
V_0	*Muzzle Velocity*	*m/s*
V(x)	*Probability that x or fewer false target elements occur in* A_S	*none*
x	*Distance*	*m*
ω	*Angle of fall*	*rad*

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